W2L1 - FIRST ORDER LINEAR MODELS

GROWTH & DECAY

Recall the initial value problem for population dynamics and radioactive decay.

$$\frac{dP}{dt} = kP \qquad \text{or} \qquad \frac{dA}{dt} = -kA$$

$$P(0) = P_0 \qquad\qquad A(0) = A_0$$

EX

Culture has P_0 # of bacteria. At $t = 1$ hour the # of bacteria is $\frac{3}{2}P_0$. If the growth rate is proportional to the # of bacteria $P(t)$ at time t, determine the time needed for the bacteria to triple.

$$\begin{cases} \dfrac{dP}{dt} = kP \\ P(0) = P_0 \end{cases} \leftarrow \frac{dP}{dt} - kP = 0 \leftarrow \text{LINEAR}$$

$$\rho = e^{\int -k\,dt} = e^{-kt}$$

$$e^{-kt}\frac{dP}{dt} - ke^{-kt}P = 0$$

$$\frac{d}{dt}\left(e^{-kt}P\right) = 0 \Rightarrow e^{-kt}P = C$$

$$P = Ce^{kt}$$

Integrating Factor Method

Use the I.C. to find C: $P(0) = Ce^{\cancel{0}} = P_0 \Rightarrow C = P_0$

$$\Rightarrow \boxed{P = P_0 e^{kt}}$$

use $P(1) = \frac{3}{2}P_0 \Rightarrow \frac{3}{2}\cancel{P_0} = \cancel{P_0}e^{k(1)} \Rightarrow \frac{3}{2} = e^{k} \Rightarrow k = \ln\frac{3}{2} \approx 0.4055$

We know $P(t) = P_0 e^{0.4055t} \Rightarrow 3P_0 = P_0 e^{0.4055t}$

$3P_0 \leftarrow$ 3 times start #

$$3 = e^{0.4055t} \Rightarrow t = \frac{\ln 3}{0.4055} = \underline{2.7093 \text{ hours}}$$